

# Detecting Network Anomalies in the Value Added Taxes (VAT) system

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- Motivation the topic and research questions
- A bit on VAT
- Description of data used in the analysis
- Description of methodology
- Results and evaluation
- Conclusion

- Research is motivated by the significant 'fraud' in Value Added Tax (VAT)
- Difficult to obtain accurate estimates—some have it that VAT fraud in EU is around 50 billion Euros (lower bound)
- Revenue Authorities do utilise algorithms, but there is scope for academic work and cooperation with such organisations
- Objective of research:
  - Develop a model which is fed with information ('and trained') to predict 'high risk' behaviour but also identify the cluster this 'high risk' behaviour belongs to (sub-network/cluster)
  - The model is applied to VAT but idea is more broadly applicable

- VAT is a **broad-based tax on consumption** and has dominated the world (as considered to be an 'efficient' tax system)
- Explicit **credit-invoice** mechanism where firms/taxable persons
  - **Levy** VAT on their output
  - **Deduct** VAT already paid on inputs, and
  - **Remit** the balance due to the government
- In one level, credit-invoice mechanism **facilitates enforcement** as it creates a **paper-trail** of transactions... but...
- Being a consumption tax, **exports are not taxable** and tax payments are subject to periodic declaration by firms
  - And this is the **Achilles' heel** of VAT—which is duly exploited by unscrupulous traders
- VAT fraud is complicated, sometimes involving dozens of firms spanning across countries/continents

- There are so opportunities for fraud... for example the Missing Trader...

## Simple Missing Trader Scheme

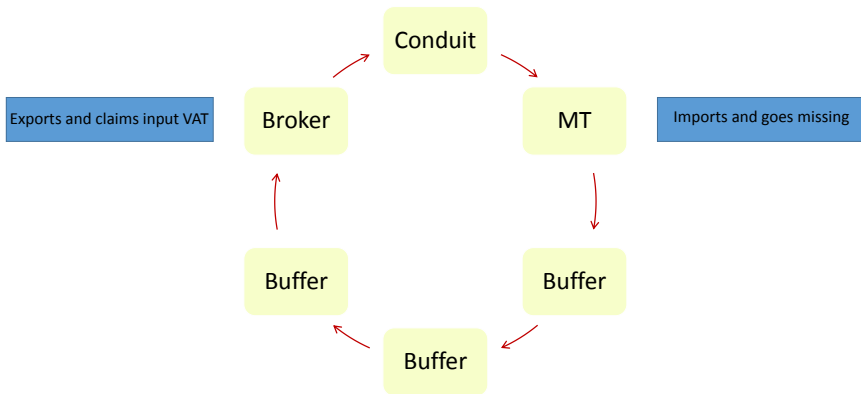
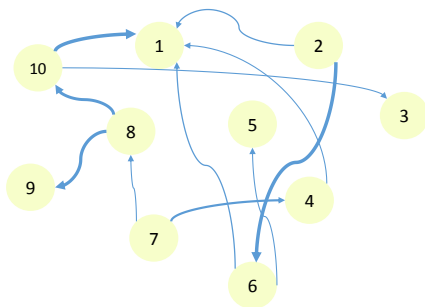


Figure 1: Missing Trade/Carousel fraud

## Revenue Authority 'sees' this Network



→ Arrow (edges) denotes direction of transactions  
Width of edges denotes size of transaction (sales in data)

Figure 2: What Revenue Agency 'Sees'



## But Real Network is this...

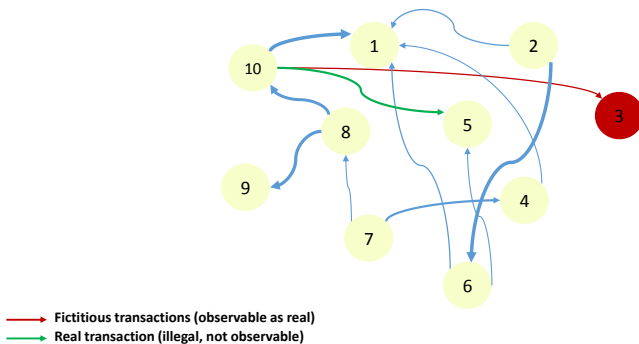
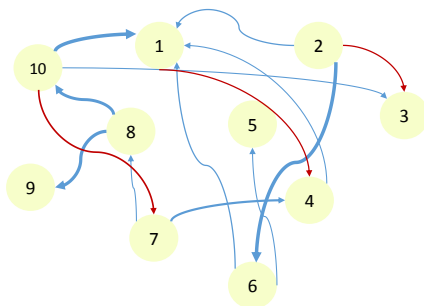


Figure 3: What Revenue Agency does not 'See'

## But it might be this the case too!

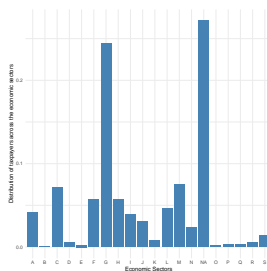
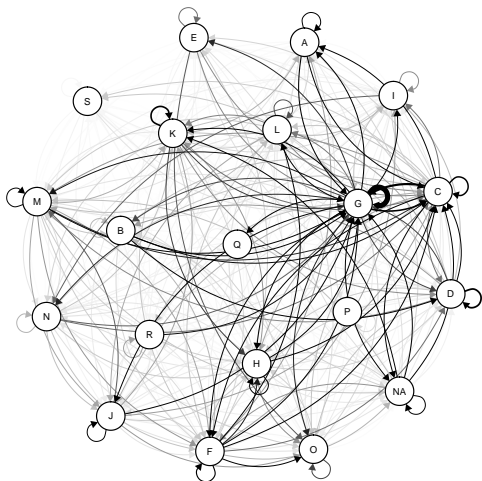


→ Fictitious transactions (not real but paper transaction—trading in 'invoices')

Figure 4: What Revenue Agency does not 'See'

- **Recover losses**...difficult [once fraud is done... it is done]
- **Disrupt** the fraud before it begins!
  - This is where we come in... through... trying to **identify** whether there are
    - 1 Particular taxpayers (vertices) **evolve irregularly** compared to the other vertices ('anomalous vertex detection'), and/or
    - 2 Groups of taxpayers (vertices) with transactions that **deviate from normal patterns** ('anomalous sub-graphs detection')

**Figure 5:** Sector-specific transactions: nodes correspond to economic sectors; edge direction represents sells



**Figure 6:** Distribution of VAT-registered traders/taxable persons across economic sectors.

- Access to the world of VAT transactions in Bulgaria involving
  - Domestic Transactions/Imports/Exports
  - Inter-community Acquisitions (I.C.A) and Deliveries (I.C.D)
  - Special acquisitions at reduced rates
  - Triangular Acquisitions (TA) and Deliveries (TD)
- VAT returns for all the monthly observed VAT transactions
  - $N = 312,762$  registered taxpayers; 75% active each month
  - 1% of taxpayers are classified as **highly risky** (criteria developed by operational knowledge at NRA and past information)
  - Average monthly transactions: 1,461,198
- Access to firm specific data: size, age of business, labour costs, sector it belongs to and the...
- Empirical probability of **risk identified** by NRA of firms in a sector

- Monthly VAT transactions are modelled as a **weighted directed graph** where
  - Each vertex/node corresponds to a VAT registered taxpayer
  - An edge between two taxpayers exists if they have exchanged at least one invoice (the direction of the edges represents sells)
  - Edge weights: The sum of the VAT base in all the sells invoices exchanged between two taxpayers

- Network notation:

- A graph is defined as  $G = (V, E)$ :  $V$  is the set of vertices (nodes) and  $E \subset V \times V$  is the set of edges
- $\mathbf{A}$  denotes the  $N \times N$  adjacency matrix of the graph

$$\mathbf{A}_{ij} = \begin{cases} w_{ij}, & \text{if } (i, j) \in E, \forall i, j \in 1, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

- $\mathbf{Y}$  denotes an  $N$ -dimensional binary vector that indicates risky taxpayers
- **Aim:** Given the monthly observed VAT networks and the vector  $\mathbf{Y}$  we want to identify individuals and groups taxpayers that perform fraudulent activity in the current month
- We work with data from January 2016 to November 2017 and we **test** the methods in detecting the fraudulent activity in December 2017

- **Proposed approach:** Utilize the available **node-specific information** (taxpayer profile) to identify **high risk taxpayers** as well as **communities of taxpayers** involved in fraudulent activities
- We develop a two-step method:
  - ① We use binary logistic regression to predict risk probabilities for each node
  - ② We employ the predicted risk probabilities to perform community detection

## Stage 1: Fraud Detection (identifying 'anomalous' nodes)

- We construct the  $N \times p$  matrix  $\mathbf{X}$  with  $p$  node-specific characteristics
- For the  $i$ th taxpayer the  $i$ th row  $X_i$  consists of:
  - Number of transactions and the corresponding VAT base within categories in Tables 1 and 2: ICA, ICD, 9%, Imports/Exports...
  - Company's size, age, time of VAT registration, labour costs, sector
  - **Number of transactions and the corresponding VAT base with highly risky taxpayers**
  - **Averages across months of the graph characteristics: in- and out-degree, in- and out- strength and centrality measures**
- We consider the data set  $\{\mathbf{X}, \mathbf{Y}\}$  to train a binary regression model by using extreme gradient boosting regression (XGboost, Chen and Guestrin, 2016)
- We use the trained regression model to obtain the  $N$ -dimensional vector  $\hat{\mathbf{Y}}$  which consists of predicted node-specific risk probabilities



- We conduct **community detection** taking into account the probabilities  $\hat{\mathbf{Y}}$
- We follow (Bienkiewicz et al., 2017) and we perform **spectral clustering** on the matrix

$$\tilde{\mathbf{L}}(\alpha) = \mathbf{L}_\tau \mathbf{L}_\tau + \alpha \hat{\mathbf{Y}} \hat{\mathbf{Y}}^T,$$

where

$$\mathbf{L}_\tau = \mathbf{D}_\tau^{-1/2} \tilde{\mathbf{A}} \mathbf{D}_\tau^{-1/2}, \quad \mathbf{D}_\tau = \mathbf{D} + \tau \mathbf{I}_N$$

- $\mathbf{D}$  is  $N \times N$  diagonal matrix where  $\mathbf{D}_{ii} = \sum_{j=1}^N \tilde{\mathbf{A}}_{ij}$
- $\tau = \frac{1}{N} \sum_{i=1}^N \mathbf{D}_{ii}$  is the average node degree and accounts for large nodes and sparse graphs
- $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{A}^T$ :
  - $\tilde{\mathbf{A}}$  is symmetric and is the adjacency of the corresponding undirected graph
  - We keep the same edges with  $\mathbf{A}$
  - Edges in both directions replaced with one weighted by their sum
- $\alpha > 0$ : tuning parameter compromising between the network structure and the probability of fraud

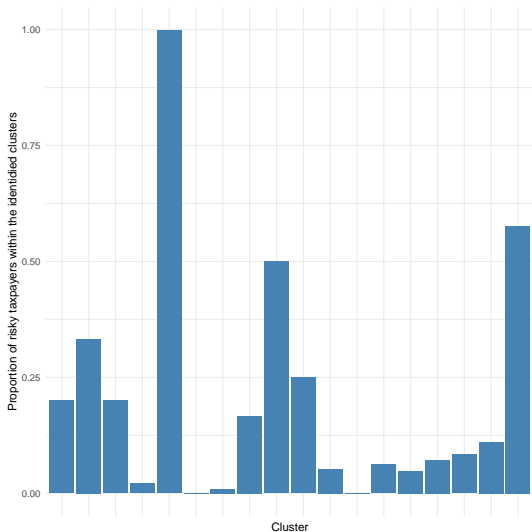
**Inputs** Graph  $G$  with  $N$  nodes,  $N \times p$  matrix  $X$  with node-specific characteristics, spectral tuning parameter  $\alpha > 0$

- 1 Run the XGboost algorithm to obtain node-specific risk probabilities  $\hat{Y}$
- 2 Construct the matrix  $\tilde{\mathbf{L}}(\alpha)$
- 3 Compute the eigendecomposition of  $\tilde{\mathbf{L}}(\alpha)$
- 4 Form the  $N \times K$  matrix  $\mathbf{U}$  with columns the eigenvectors of the  $K$  largest eigenvalues
- 5 Normalize each row in  $\mathbf{U}$  to have unit length
- 6 Treat each normalized row of  $\mathbf{U}$  as point in  $\mathbb{R}^K$  and run a  $k$ -means clustering algorithm with  $K$  clusters
- 7 If the  $i$ th row of  $\mathbf{U}$  falls in the  $k$ th cluster assign node  $i$  to cluster  $k$

**Outputs**  $K$  clusters which include the nodes of the graph  $G$ , node-specific risk probabilities

- We identified  $K = 191$  clusters with at least two members in each one
  - 70% of the identified clusters had 10 or less members
  - 25% of the clusters have size between 10 and 100
  - 5 clusters with more than 100 members but less than 1,000
- The largest cluster contains 94% of the VAT registered taxpayers:
  - Includes only 200 out of 2,192 taxpayers marked as high risk by the authorities
  - We consider this as the cluster with legitimate taxpayers
  - This implies less than 10% rate of false negatives
- The remaining 190 clusters have in total 10,624 taxpayers; 2,016 of them already identified from the authorities implying 92% true positive rate of our method

# Results: Identifying known fraudsters and testing the method



**Figure 7:** Proportion of VAT registered taxpayers persons that are already identified by the tax authorities as non-legitimate within each cluster. We display the proportions for the 18 clusters which include at least one non-legitimate taxpayer.

- 8,608 taxpayers in the 190 clusters have not been identified as non-legitimate from the authorities
- We choose 35 (practical restrictions) to be further investigated from the authorities as follows:
  - We rank the 8,608 taxpayers by using the **predicted node-specific risk probabilities** and we select the first 10
  - To select 15 more we **rank the clusters** that contain at least one known fraudster **by using the mean risk probability within each cluster**; we choose the 15 first clusters and from each one we select the taxpayer with the highest risk probability
  - We select the last 10 by following the same procedure for clusters but consisted completely of **unknown fraudsters**
- Tax authority has reported that 12 out of the 35 VAT-registered traders/taxable considered as high risk (but not £value has been given)

- VAT fraud is significant
- Project has developed a method that identifies clusters of fraudulent transactions
- Limitation: Characterisation of size of fraud across clusters is needed as Revenue Authorities are capacity constrained (we are working on this)

Thank you for listening!

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