Engineering and Physical Sciences Research Council Doctoral Landscape Award

PROJECT TITLE: Skew Braces via Transitive Subgroups

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Co-Supervisors: Prof Henri Johnston

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Project details: The Yang-Baxter equation (YBE) first arose in physics, but has connections to many topics in mathematics, including quantum groups and knot theory. The search for set-theoretical solutions of the YBE has led to the introduction of various new algebraic objects. Among these are braces [5] and skew braces [3]. A skew brace is a set which is a group in two different ways (we call the resulting groups the additive group and multiplicative group) satisfying a certain compatibility condition. It is a brace if the additive group is abelian. For a given group N, finding skew braces with additive group N is equivalent to finding regular subgroups in the holomorph of N (that is, the semidirect product of N with its automorphism group). This is precisely the same problem as arises in Hopf-Galois theory, which generalises classical Galois theory by replacing the Galois group by a Hopf algebra. Any Hopf-Galois structure on a Galois extension of fields L/K corresponds to an embedding of the Galois group Gal(L/K) as a regular subgroup of the holomorph of some group N. More generally, transitive subgroups of the holomorph of N correspond to Hopf-Galois structures on field extensions which are not necessarily normal. An analogous generalisation of skew braces is given by skew bracoids [4].

The aim of this project is to investigate some open problems in the theory of skew braces from the distinctive perspective provided by Hopf-Galois theory. We describe two of these problems below:

(1) Does there exist a finite skew brace with soluble additive group and insoluble multiplicative group? (There are examples the other way round.) It has been shown, using a consequence of the classification of finite simple groups, that in a minimal counterexample, the only insoluble composition factor is the simple group of order 168 [2]. One area for investigation is whether one can obtain further structural information on such counterexamples, with a view to either constructing one or showing that one cannot exist.

(2) Which finite groups occur as the multiplicative groups of braces (i.e. with additive abelian groups)? Such groups are called Involutive Yang-Baxter (IYB) groups. It is known that any IYB group is soluble, and that certain (fairly exotic) p-groups are not IYB groups [1]. A natural question is then whether a finite soluble group, all of whose Sylow subgroups are IYB groups, is itself an IYB group. A reinterpretation of this in terms of regular subgroups of holomorphs of abelian p-groups can be expected to produce new insights into this question.

[1] D. Bachiller: Counterexample to a conjecture about braces, J. Algebra 453 (2016), 160-176.
[2] N. Byott: On insoluble transitive subgroups in the holomorph of a finite soluble group. J. Algebra 638 (2024), 1-31. [3] L, Guarnieri, L. Vendramin: Skew braces and the Yang-Baxter

equation: Math. Comp. 86 (2017), 2519-2534 [4] I. Martin-Lyons, P. Truman: Skew bracoids, J. Algebra 638 (2024), 751-787 [5] W. Rump: Braces, radical rings, and the quantum Yang-Baxter equation, J. Algebra 307 (2007) 15-170

Project Specific requirements: Strong undergraduate background in pure mathematics

Potential PhD programme of study: PhD in Mathematics

Department: Mathematics and Statistics

Location: Harrison, Streatham

Please direct project specific enquiries to: Prof. Nigel Byott, N.P.Byott@exeter.ac.uk

Please ensure you read the entry requirements of programme to which you are applying.

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