Prospect Theory and Tax Evasion: A Reconsideration

Amedeo Piolatto   Matthew D. Rablen

July 2013

Introduction

- Yitzhaki puzzle: The expected utility model of tax evasion predicts a negative relationship between tax rates and evasion when preferences satisfy DARA. Most empirical evidence finds the opposite.
- Recent years have seen several attempts to employ the insights of prospect theory to the tax evasion decision
  - These include Bernasconi and Zanardi (2004), Dhami and al-Nowaihi (2007), Trotin (2012) and Yaniv (1999).
- This literature is reviewed by Hashimzade, Myles and Tran-Nam (in press).

Literature Claims

- Dhami and al-Nowaihi (2007: 171) claim to "...show that prospect theory provides a much more satisfactory account of tax evasion including an explanation of the Yitzhaki puzzle."
- Hashimzade et al. (in press: 16) conclude (on the basis of several examples) that "Prospect theory does not necessarily reverse the direction of the tax effect: our examples show that certain choices of the reference level can affect the direction of the tax effect in some situations, but none of the examples is compelling."
- We investigate this dichotomy.

Our Contribution

- We revisit the tax evasion model under expected utility theory and under various reference dependent models.
- We allow R to be a (general) decreasing function of the marginal tax rate.
- We analyze the model both with the probability of audit
  - fixed exogenously;
  - as a function of the taxpayer's declaration.
- We find that
  - There are clear-cut versions of prospect theory that reverse the Yitzhaki puzzle.
  - Prospect theory does not reverse the Yitzhaki puzzle for existing psychologically plausible specifications of the reference level.

Prospect Theory

- Prospect theory bundles four key elements
  - *reference dependence*: outcomes judged relative to a reference level of wealth R: this may be exogenously or endogenously specified
  - *diminishing sensitivity*: marginal utility is diminishing in distance from the reference level
  - *loss aversion*: the disutility of a loss exceeds the utility of a gain of equal magnitude
  - *probability weighting*: objective probabilities transformed into decision weights
- Previous literature has not spelled out which of these concepts are needed for particular results
- We analyse the effects of these elements separately and in combination

Notation

- v is taxpayer utility, and $v' > 0$.
- Y = exogenous taxable income (which is known by the taxpayer but not by the tax authority).
- The government levies a proportional income tax at marginal rate t on declared income X.
- The probability of audit is $p \in (0, 1)$.
- Audited taxpayers face a fine at rate $f > 1$ on all undeclared tax.
- Wealth when the taxpayer is caught (audited) and when not caught are therefore

$$Y^n = Y - tX; \qquad Y^c = Y^n - tf\,(Y - X)\,.$$

Fixed p and Expected Utility

- Assume $v'' < 0$.
- Expected utility given by $V = pv(Y^c) + (1-p)\,v(Y^n)$
- First order condition

$$\frac{\partial V}{\partial X} = t\left(p\,(f-1)\,v'(Y^c) - (1-p)\,v'(Y^n)\right) = 0.$$

- Second order condition

$$\frac{\partial^2 V}{(\partial X)^2} = D = t^2\left\{p\,(f-1)^2\,v''(Y^c) + (1-p)\,v''(Y^n)\right\} < 0.$$

Fixed p and Expected Utility

- The derivative $\partial X / \partial t$ is derived as

$$\frac{\partial X}{\partial t} = -\frac{-t\left(p\left(f-1\right)\left(X+f\left(Y-X\right)\right)v''\left(Y^c\right)-\left(1-p\right)Xv''\left(Y^n\right)\right)}{D}. \tag{1}$$

- Adding and subtracting $t^{-1}D\left(Y-X\right)$ in the numerator, and applying the first order condition, (1) rewrites as

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left\{\left(Y-X\right)-\frac{Y\left\{A\left(Y^n\right)-A\left(Y^c\right)\right\}}{\left(f-1\right)A\left(Y^c\right)+A\left(Y^n\right)}\right\}.$$

where $A\left(x\right) = -v''\left(x\right)/v'\left(x\right)$ is the Arrow-Pratt coefficient of absolute risk aversion.

Fixed p and Expected Utility

Proposition

(Yitzhaki, 1974) At an interior maximum, $\partial X / \partial t > 0$.

- Result is a pure wealth effect: θ increases $\longrightarrow$ taxpayers feel poorer $\longrightarrow$ taxpayers become more risk averse

Fixed p and Exogenous Reference Dependence

- An "exogenous" reference level is one taken to be independent of both X and t (but which could, e.g., be a function of Y).
- The objective function is $V_R = pv\left(Y^c - R\right) + (1-p)\, v\left(Y^n - R\right)$.
- Repeating the steps as in the expected utility model above we obtain

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left\{(Y - X) - \frac{Y\left\{A\left(Y^n - R\right) - A\left(Y^c - R\right)\right\}}{(f-1)\, A\left(Y^c - R\right) + A\left(Y^n - R\right)}\right\}.$$

Proposition

At an interior maximum, $\partial X / \partial t > 0$.

- Loss aversion is already implied in this model, for $-v\left(-x\right) > v\left(x\right)$ for $x > 0$ by the strict concavity of v.
- Adding probability weighting (by replacing p with $w\left(p\right)$) leaves the result unchanged.

Introducing Diminishing Sensitivity

- Replace $v(x)$ with $v^{-}(x)$ for $x < 0$, where $v^{-\prime\prime} > 0$ such that $A^{-}(x) < 0$.
- No guarantee that an interior maximum exists or is unique.
- We focus on the only interesting case: $Y^{n} > R > Y^{c}$.
- Objective function is $V_{DS} = pv^{-}(Y^{c} - R) + (1-p)\, v(Y^{n} - R)$.
- We obtain

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left\{(Y - X) - \frac{Y\left\{A(Y^{n} - R) - A^{-}(Y^{c} - R)\right\}}{(f-1)\, A^{-}(Y^{c} - R) + A(Y^{n} - R)}\right\}.$$

Proposition

At an interior maximum satisfying $Y^{n} > R > Y^{c}$, $\partial X/\partial t < 0$.

- Result is a pure wealth effect: θ increases $\rightarrow$ taxpayers feel poorer $\rightarrow$ taxpayers become *less* risk averse

Fixed p and $R = R(t)$

- Assume $R_t < 0$, $R_X = 0$.
- Most popular specification is $R = Y(1-t)$, so $R_t = -Y$.
- The derivative $\partial X / \partial t$ now becomes

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left\{(Y-X) - \frac{(Y+R_t)\left\{A(Y^n - R) - A(Y^c - R)\right\}}{(f-1)A(Y^c - R) + A(Y^n - R)}\right\}.$$

- Assuming diminishing sensitivity we obtain

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left\{(Y-X) - \frac{(Y+R_t)\left\{A(Y^n - R) - A^-(Y^c - R)\right\}}{(f-1)A^-(Y^c - R) + A(Y^n - R)}\right\}$$

- Immediately apparent that $R_t = -Y$ does not reverse Yitzhaki puzzle.
- Moreover, this holds whether we use $A(\cdot)$ or $A^-(\cdot)$.

Fixed p and $R = R(t)$

Proposition

Assume $R_t < 0$ *and* $R_X = 0$. *Then:*
(i) assuming DARA, there exists a threshold level $\widetilde{R}_t < -Y$ *such that, at an interior maximum,* $\partial X / \partial t < 0$ *for* $R_t < \widetilde{R}_t$ *and* $\partial X / \partial t \geq 0$ *for* $R_t \geq \widetilde{R}_t$.
(ii) assuming diminishing sensitivity, there exists a threshold level $\widetilde{R}_{t,DS} > -Y$ *such that, at an interior maximum,* $\partial X / \partial t < 0$ *for* $R_t > \widetilde{R}_{t,DS} > -Y$ *and* $\partial X / \partial t \geq 0$ *for* $R_t \leq \widetilde{R}_{t,DS}$.
(iii) parts (i) and (ii) hold if loss aversion and/or probability weighting are additionally assumed.

Intuition

- Without diminishing sensitivity
  - Model requires reference level to be sufficiently *sensitive* to t.
  - Then θ increases $\rightarrow$ expected wealth increases (R falls faster than expected value of the tax gamble) $\rightarrow$ taxpayers feel richer (relative to the reference level) $\rightarrow$ taxpayers become *less* risk averse
- Without diminishing sensitivity
  - Model requires reference level to be sufficiently *insensitive* to t.
  - θ increases $\rightarrow$ expected wealth falls, and R falls slower than expected wealth $\rightarrow$ taxpayers feel poorer (relative to the reference level) $\rightarrow$ taxpayers become *less* risk averse

Corollary

Corollary

Assume endogenous reference dependence, $R_X = 0$*, and* $R_t \in \left(-\widetilde{R}_h, -\widetilde{R}_l\right)$*. Then, at an interior maximum,* $\partial X / \partial t > 0$ *whether or not diminishing sensitivity is assumed.*

- Corollary implies Yaniv's (1999) result holds only under further (and strong) restrictions, and that Proposition 8 of Trotin (2012) is false.

Fixed p and $R = R(t,X)$

- Assume $R_t < 0$, $R_X < 0$, R_X homogeneous of degree one in X
- The expected value satisfies these properties, as does $R = (1 - t)\,X$

Proposition

Assume $R_t < 0$, $R_X < 0$, $R_{XX} = 0$ and R_X homogeneous of degree one in t. Then parts (i)-(iii) of Proposition 2 hold unchanged, and so does its Corollary

Endogenous p

- Let $p = p(X)$.
- Few, if any, general results hold. Instead we focus on the setting employed in Dhami and al-Nowaihi (2007).
- These authors employ a power function for v (implying homogeneity) and $R = Y(1-t)$.

Proposition

Assume endogenous reference dependence, v homogeneous, $p'(X) \leq 0$ and $R = Y(1-t)$. Then, at an interior maximum, $\partial X / \partial t = 0$.

- Hence, allowing for prospect theory and/or $p'(X) < 0$ does not resolve Yitzhaki's puzzle in this model.
- Key to result:
  $V_{p(X)} = v(t)\, v(Y - X) \{p(X)\, v(-(f-1)) + 1 - p(X)\}$
- What does explain Dhami and al-Nowaihi's finding?

Stigma and Prospect Theory

- Dhami and al-Nowaihi introduce a stigma parameter that such that wealth when caught becomes

$$Y^c = Y - tX - (s + ft)(Y - X).$$

Proposition

(Dhami and al-Nowaihi, 2007) Assume endogenous reference dependence, stigma, v homogenous of degree $\beta > 0$, $p' \leq 0$, and $R = Y(1 - t)$. Then, at an interior maximum, $\partial X / \partial t < 0$.

- So prospect theory combined with stigma reverses the Yitzhaki puzzle.

Stigma and Expected Utility Theory

- Is stigma also able to reverse the Yitzhaki puzzle when combined with expected utility theory?

Proposition

Assume expected utility theory, stigma, $p' < 0$, and risk neutrality. Then, at an interior maximum, $\partial X / \partial t < 0$.

- So stigma reverses the Yitzhaki puzzle in both the expected utility and reference-dependent models.

Conclusions

- Prospect theory robustly reverses the Yitzhaki puzzle for an exogenous reference level. Hence it cannot be written-off as an explanation of the Yitzhaki puzzle.
- But existing analyses with an endogenous reference level fail to reverse the puzzle. In particular, the reference dependent model cannot reverse the Yitzhaki puzzle around $R_t = -Y$, irrespective of the shape of v.
- Probability weighting and loss aversion are inessential features of prospect theory in respect of reversing the Yitzhaki puzzle: reference dependence and diminishing sensitivity are the only two concepts required.